EXAM COMPUTER VISION, INMCV-08

April 7, 2015, 18:30-21:30 hrs

□—□—□—□—□—□—□—□—□—□—□—□—□—□—□—□—□—□—□

During the exam you may use the lab manual, copies of sheets, **provided they do not contain any notes.**

Put your name on all pages which you hand in, and number them. Write the total number of pages you hand in on the first page. Write clearly and not with pencil or red pen. ***Always motivate your answers.*** *The total number of points to be earned is 9, one is added for free. Good luck!*

Problem 1. (2.0 pt) Consider a surface centred at the origin with equation

$$z(x,y) = d - x^2 - y^4$$

The surface is Lambertian with constant albedo $\rho_S = 1$, and is illuminated by a light source at a very large distance, from a direction defined by the unit vector $\vec{s} = (a,b,c)^T$, with c negative. The camera is on the negative z-axis.

a. (1.0pt) Determine the image intensity $E(x,y)$ under orthographic projection ($u = x, v = y$).

b. (0.5pt) Suppose $\vec{s} = (1,0,0)^T$, i.e., the light source is in the direction of the positive x-axis. What is the observed light intensity $E(x,y)$ for $x < 0$ according to the equation for $E(x,y)$ you derived? How should we interpret this?

c. (0.5pt) Given $E(x,y)$, can we reconstruct the surface function $z(x,y)$? If not: what is missing and what could we do to resolve the problem?

Problem 2. (2.5pt) A linear scale space can be implemented as a convolution with Gaussian kernel

$$K_\sigma(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}. \quad (1)$$

of varying standard deviation σ. Thus, the equation

$$I_t(x,y) = \mathcal{S}^t I(x,y) = I_0(x,y) * K_{\sqrt{2t}}(x,y) \quad (2)$$

defines the scale space, with $I_0(x,y)$ the original image. In the following assignment, assume as given the following expression for a convolution of two Gaussian functions:

$$K_{\sigma_1}(x,y) * K_{\sigma_2}(x,y) = \frac{1}{2\pi(\sigma_1^2+\sigma_2^2)} e^{-\frac{x^2+y^2}{2(\sigma_1^2+\sigma_2^2)}}, \quad (3)$$

and, recall that a convolution with any kernel defines a linear, shift-invariant system. Consider the six axioms of scale spaces.

a. (0.5 pt) Argue that the linear scale space is translation invariant

b. (0.5pt) Argue that the linear scale space is rotation invariant

c. (1.0pt) Show that this scale space is causal in the sense of

$$\mathcal{S}^t \mathcal{S}^s I = \mathcal{S}^{t+s} I, \qquad \forall s,t \geq 0.$$

d. 0.5pt Show that this scale space is contrast invariant in the sense that

$$\mathcal{S}^t(gI) = g(\mathcal{S}^t I).$$

with g an arbitrary constant.

Problem 3. (2.5 pt) Suppose a camera sees cars approaching on an intersection, and through tracking features on the cars detects linear motion with centres of expansion or contraction $(u_1, v_1) = (-25, 0)$ and $(u_2, v_2) = (25, 0)$, respectively.

a. (1.0 pt) Compute the normal to the plane in which the intersection lies as a unit vector in camera-centric coordinates (z-direction along optical axis). Do you need to know the camera constant f?

b. (1.0 pt) Express the angle α between the roads at the intersection as a function of (unknown) camera constant f.

c. (0.5 pt) What is f, assuming the roads cross at right angles?

Problem 4. (2.0 pt) Consider an image E with grey level distribution given by

$$E(u_0, v_0) = A \sin(\omega u_0) \tag{4}$$

at $t = 0$ (u_0 and v_0 denote position at $t = 0$), with ω the spatial frequency of the pattern, and A the amplitude. The motion field in the image plane is given by

$$\dot{u} = 1, \quad \dot{v} = 2. \tag{5}$$

a. (0.5 pt) Give an expression for $E(u, v, t)$ (**Hint:** first obtain expressions for $u(t)$ and $v(t)$).

b. (0.5 pt) Compute the observed temporal changes in irradiance $\frac{\partial E}{\partial t}$ as a function of u and v (**Hint:** if you did not solve part a., use the Horn-Schunck equation).

c. (1.0 pt) Given the temporal changes (optic flow) obtained in part b., can the motion field be recovered completely using the Horn-Schunck equation? If not, would addition of smoothness constraints solve this problem? Explain your answer.